

Toward a standard model 2, via Kaluza ansatz 2

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Abstract

New results and perspectives precipitate from the (modified as) Kaluza ansatz 2 (KA2), whereby, instead of appending n Planck-scale (L_o) compact SL dimensions to ordinary 4D spacetime, one *augments* n such dimensions by 3 large ones. By KA2, the fundamental rôle of gravity in the dynamics of vacuum geometry is being conceded to the remaining fundamental interactions. The ground state in KA2 is of the form $\bar{\mathcal{M}}^{n+4} = \bar{\mathcal{C}}^{n+1} \times \mathbb{R}^3$, where the static (averaged-out over scales $L \gg L_o$) $\bar{\mathcal{C}}^{n+1}$ carries *effective torsion* as relic of the deeper vacuum dynamics at Planck scale. For the simplest non-trivial implementation of KA2, the Bianchi IX subclass of $SU(2)$ -invariant \mathcal{B}_{IX}^4 provides the $\bar{\mathcal{C}}^5 = \bar{\mathcal{B}}_M^4 \times S^1$, with the S^1 coming from ‘augmentability’, a complement to compactification. The classical action involves (i) the gravitational and EW sectors in elegant *hierarchy*, (ii) the *higgsless* emergence and full calculability of the EW gauge bosons masses and (iii) gravity as a necessarily effective field, hence non-quantizable. A conjectured \mathcal{C}^{n+1} with $n \leq 7$ (to adjoin the strong interaction) toward a standard model 2, might also offer novel perspectives for supergravity.

Keywords: Kaluza-Klein theories, Taub string, hierarchy, torsion, higgsless EW masses, compactification, augmentability, standard model, supergravity, quantization of gravity.

1 Introduction

The main task at LHC may be impeded by the Higgs sector of the standard model, but the latter will require even deeper reform, if the former folds at LHC, because of its other fundamental problems, notably on hierarchy and the quantization of gravity. Curiously related, forty years before the collective formulation of the Higgs mechanism, the Kaluza ansatz was likewise received as a ‘clever artifact’ (for the enlargement of the then young theory of general relativity), to be likewise elevated subsequently to a fundamental notion, but its geometric elegance has remained unquestionably unique all along. The fundamental interactions can be segregated by physical aspects (dimension-less vs -full couplings and quantization) but not by *a priori* geometrical ones in a unified higher-dimensional context. Nevertheless, one may resort to the *approach* towards full geometrization via the standard Kaluza ansatz [1], for a complementary KA2 approach, in the sense that, instead of appending n Planck-scale (L_o) compact SL dimensions to ordinary general-relativistic 4D spacetime, one can *augment* n such dimensions of a \mathcal{C}^{n+1} proper vacuum [2] by 3 large ones.

By KA2, the central rôle of gravity in the dynamics of vacuum geometry is being conceded to the remaining fundamental interactions. The ground state in KA2 is of the form $\bar{\mathcal{M}}^{n+4} = \bar{\mathcal{C}}^{n+1} \times \mathbb{R}^3$, where the static (averaged-out over scales $L \gg L_o$) $\bar{\mathcal{C}}^{n+1}$ carries *effective torsion* [2], [3] as relic of the deeper vacuum dynamics at Planck scale. The requirement of *augmentability* (a complement to that of spontaneous compactification) will curtail the already-small class of \mathcal{C}^{n+1} . The latter can be a homogeneous space [4], [5] with vigorous or even chaotic dynamics, like Misner’s elusive mixmaster \mathcal{B}_M^4 [6] in the Bianchi IX subclass of left- $SU(2)$ invariant \mathcal{B}_{IX}^4 , which also includes the Taub string [2], a pp wave \mathcal{B}_T^4 . In the static $\bar{\mathcal{B}}_M^4$, $\bar{\mathcal{B}}_T^4$, the effective torsion \bar{T}^A (parallelizing in the second case) is explicitly calculable via the effective loss of the Ricci flatness in \mathcal{B}_M^4 , \mathcal{B}_T^4 [2]. The simplest possible non-trivial ground state for K-A2 involves a $\bar{\mathcal{M}}^8$ with a $\bar{\mathcal{C}}^5 = \bar{\mathcal{B}}_M^4 \times S^1$, where $\bar{\mathcal{B}}_M^4$ is chosen for its round rather than squashed S^3 ; the S^1 factor is actually *imposed* by augmentability, as we will see, whereby the topology of the SL sections and the transitive $SU(2)$ invariance on them must be enlarged to at-least $S^3 \times S^1$ and $SU(2) \times U(1)$, respectively. For a comparative view, we will also cite the standard M_o^8 ground state (with Minkowski’s M_o^4) as depicted in the schemes

$$M_o^8(\bar{e}^A, \bar{\Gamma}^A_B) := M_o^4 \times (S^3 \times S^1) \xrightarrow{\delta \bar{e}} M^8(e^A, \Gamma^A_B), \quad (1.1)$$

$$\bar{\mathcal{M}}^8(\bar{e}^A, \bar{\gamma}^A_B) := (\bar{\mathcal{B}}_M^4 \times S^1) \times \mathbb{R}^3 \xrightarrow{\delta \bar{e}, \delta \bar{\gamma}} \mathcal{M}^8(e^A, \gamma^A_B), \quad (1.2)$$

for the standard vs the KA2 approach. The difference between M_o^8 and $\bar{\mathcal{M}}^8$ may at first appear to be rather trivial, because they both have the same topology and metric (or \bar{e}^A Cartan frames), hence also identical $\bar{\Gamma}^A_B$ Christoffel connections, so their only difference is

the presence of the effective torsion \bar{T}^A in the $\bar{\gamma}_B^A$ connection of $\bar{\mathcal{M}}^8$. Nevertheless, their difference in perspective and results will turn out to be fundamental. In either case we arrive at a ‘low-energy’ configuration, the $M^8(e^A, \Gamma_B^A)$, $\mathcal{M}^8(e^A, \gamma_B^A)$, respectively.

In the standard case, the process of starting with the $M_o^8(\bar{e}^A, \bar{\Gamma}_B^A)$ ground state to arrive by (1.1) at $M^8(e^A, \Gamma_B^A)$ is geometrically viewable as a tilt from excitation of the frames from their value \bar{e}^A in M_o^8 to $e^A = \bar{e}^A + \delta\bar{e}^A$ in $M^8(e^A, \Gamma_B^A)$. The physical content of this excitation is, of course, the $SU(2) \times U(1)$ gauge-field potentials \mathcal{A}^I , which, in the case of (1.2) for the KA2 approach, must *also* excite the torsion \bar{T}^A in $\bar{\mathcal{M}}^8$. By the holonomy theorems and the Cartan structure equations for any (\mathcal{R}_B^A, T^A) set [3], the torsion T^A (field-content *and* scale) is completely independent from the Riemannian part R_B^A of the curvature \mathcal{R}_B^A . Thus, excitations under the KA2 approach to reach $\mathcal{M}^8(e^A, \gamma_B^A)$ in (1.2) must be of the ‘metric *and* connection’ Palatini type, namely independent excitations of frames and of torsion, so they will necessarily involve (beyond κ_o, L_o) two new independent scales, the κ and L_1 , respectively. Classically they can be of virtually any amplitude, limited only by the strength κ_o^{-2} of the Taub-string, which is of Planck scale, and likewise for \mathcal{B}_M^4 . However, as with the otherwise stable Minkowski vacuum in the standard approach, the addition ‘by-hand’ of *any* mass in \mathcal{B}_T^4 or \mathcal{B}_M^4 would cause a mathematical singularity [5]. As long as this cannot be averted by the overlying torsion, mass terms in the respective actions can be generated only *effectively* by the geometry, or the vacuum stability will be lost.

Notes on notation: The indices $A, B, M \dots$ run as $M = (\mu; m) = (0, 5, 6, 7; 1, 2, 3, 4)$ with (1,2,3,4) in the compact dimensions. In all our Cartan frames and duals (e^M, E_N) we employ orthonormal $\eta_{AB} = \text{diag}(-1, +1, \dots, +1)$ and $I = (i, 4) = (1, 2, 3, 4)$ indices for the $SU(2) \times U(1)$ left-invariant 1-forms with $d\ell^i = -\frac{1}{2}\epsilon_{jk}^i \ell^j \ell^k$, $d\ell^4 = 0$. Due to isometries on $S^3 \times S^1$ there exist four transitive Killing vectors Ξ_I and their components Ξ_I^m remain invariant under both types of the Kaluza ansatz. Commutation relations between the L_μ, Ξ_ν and Lie derivatives \mathcal{L}_{Ξ_I} (by use of the duality relation $\ell^m(L_n) = \delta_n^m$, etc.) can be summarized as [1]

$$[L_j, L_m] = \delta_m^k \epsilon_{jk}^i L_i, \quad [\Xi_j, \Xi_m] = \delta_m^k \epsilon_{jk}^i \Xi_i, \quad \mathcal{L}_{\Xi_I} L_m := [\Xi_I, L_m] = 0, \quad \mathcal{L}_{\Xi_I} \ell^m = 0. \quad (1.3)$$

The general connection γ_N^M and the Christoffel $\Gamma_N^M = \Gamma_{NP}^M e^P$ in the covariant derivatives \mathcal{D}, D , respectively, are antisymmetric in M, N just like the contorsion tensor-valued 1-form K_N^M in $\gamma_N^M = \Gamma_N^M + K_N^M$ with $De^M := de^M + \Gamma_N^M \wedge e^N \equiv 0$, $DE_M = dE_M - \Gamma_N^M E_N \equiv 0$. The general curvature \mathcal{R}_B^A includes its Riemannian part $R_B^A := d\Gamma_B^A + \Gamma_P^A \wedge \Gamma_B^P$, with the Weyl and Ricci tensors W_{BMN}^A and $R_{MN} = R_{MPN}^P$. Cartan’s first and second structure equations involve the general curvature \mathcal{R}_N^M and the torsion T^M 2-forms as [3]

$$\mathcal{R}_B^A : = d\gamma_B^A + \gamma_P^A \wedge \gamma_B^P = R_B^A + DK_B^A + K_P^A \wedge K_B^P = \frac{1}{2} \mathcal{R}_{BNP}^A e^N \wedge e^P, \quad (1.4)$$

$$T^M : = \mathcal{D}e^M = de^M + \gamma_N^M \wedge e^N = K_N^M \wedge e^N = \frac{1}{2} T_{NP}^M e^N \wedge e^P. \quad (1.5)$$

2 Tilting the frames in $\bar{\mathcal{M}}^8(\bar{e}^A, \bar{\gamma}^A_B)$ towards $\mathcal{M}^8(e^A, \gamma^A_B)$

To implement K-A2, we must fix the frames etc for $\bar{\mathcal{M}}^8(\bar{e}^A, \bar{\gamma}^A_B)$ in (1.2), then proceed with the tilt $e^A = \bar{e}^A + \delta\bar{e}^A$ in terms of \mathcal{A}^I and (in the next section) with the excitation $\delta\bar{T}^A$. From $(\bar{e}^M; \bar{E}_N) = (\bar{e}^\mu = \delta^\mu_{\bar{\mu}} dx^{\bar{\mu}}, \bar{e}^m = L_o \ell^m; \bar{E}_\nu = \delta^\nu_{\bar{\nu}} \partial_{\bar{\nu}}, \bar{E}_n = L_o^{-1} L_n)$, with trivial vierbeins $\delta^\mu_{\bar{\mu}}$ for holonomic \bar{e}^μ , we find $\bar{\gamma}^A_B = \bar{\Gamma}^A_B + \bar{K}^A_B$ and the non-vanishing $\bar{\gamma}^i_j = 2\bar{\Gamma}^i_j = 2\bar{K}^i_j = \epsilon^i_{jk} \bar{e}^k$; the Ricci and scalar contractions from the Riemannian part (\bar{R}^i_{jkl}) of the full curvature $(\bar{\mathcal{R}}^i_{jkl})$ are $\bar{R}_{ij} = 1/2 L_o^{-2} \eta_{ij}$, $\bar{R} = 3/2 L_o^{-2}$, identical to those of $M_o^8(\bar{e}^A, \bar{\Gamma}^A_B)$ in (1.1). For the Riemann-Cartan geometry in $\bar{\mathcal{M}}^8(\bar{e}^A, \bar{\gamma}^A_B)$ of (1.2), the parallelizing torsion gives a $\bar{K}^i_j = 1/2 \epsilon^i_{jk} \bar{e}^k$ in $\bar{\mathcal{B}}^4_M$, hence $\bar{\mathcal{R}}^i_{jkl} = 0$. The vanishing of the Hilbert-Einstein-Cartan Lagrangian $\bar{\mathcal{L}}_{\text{HEC}} = \bar{\mathcal{R}}$ for $\bar{\mathcal{M}}^8$ offers harmless simplicity until (4.1). The orthonormality relations between the Killing vectors Ξ_I can be expressed in terms of a continuous angle parameter $\theta \in (0, \pi/2)$, the *slicing angle* θ (to be distinguished from Weinberg's *mixing angle*¹ θ_W), as

$$\Xi_I^m \Xi_J^n \eta_{mn} := \left(\frac{L_o}{\sin \theta} \right)^2 \eta_{ij} \delta_I^i \delta_J^j + \left(\frac{L_o}{\cos \theta} \right)^2 \eta_{44} \delta_I^4 \delta_J^4. \quad (2.1)$$

The scale L_o of the components is imposed by the frames \bar{e}^A and the lengths $L_o/\sin \theta$, $L_o/\cos \theta$ are proportional to the radii of S^3 and S^1 in any particular slicing of the $S^3 \times S^1$ torus, as fixed by θ . These Ξ_I provide a basis for tangent vectors on $S^3 \times S^1$, just like the L_m do. However, while the L_m are *ab initio* left-invariant, by $\mathcal{L}_{\Xi_I} L_m = 0$ etc., the Ξ_I cannot possibly form a left-invariant basis, due to the $\mathcal{L}_{\Xi_I} \Xi_J \neq 0$ relations from (1.3). Therefore, under ordinary circumstances, the Ξ_I would be an odd and cumbersome (albeit fully legitimate) basis to employ in left-invariant environments, such as those involving the round or even the squashed S^3 . This observation will be useful to us later on.

The excitation of the frames in $e^A = \bar{e}^A + \delta\bar{e}^A$ (etc., via $\ell^m(L_n) = \delta_n^m$) in KA2 is linear in the gauge potentials \mathcal{A}^I and identical to that of the standard case as

$$\bar{e}^A \rightarrow e^A := \bar{e}^A + g [\Xi \cdot \mathcal{A}]^m \delta_m^A \iff \bar{E}_B \rightarrow E_B = \bar{E}_B - g [\Xi \cdot \mathcal{A}]_\nu \delta_B^\nu, \quad (2.2)$$

where g is a scaleless coupling parameter² and the potentials enter through the components of the diagonal tensor $[\Xi \cdot \mathcal{A}]$ of mixed (1,1) rank (to be discussed shortly) defined as

$$[\Xi \cdot \mathcal{A}]^m := \Xi_i^m \mathcal{A}^i \sin \theta + \Xi_4^m \mathcal{A}^4 \cos \theta, \quad [\Xi \cdot \mathcal{A}]_\nu = \Xi_i \mathcal{A}^i_\nu \sin \theta + \Xi_4 \mathcal{A}^4_\nu \cos \theta. \quad (2.3)$$

The transformations in (2.2) can be viewed as trivially reversible with the simple transfer of the terms involving $g[\Xi \cdot \mathcal{A}]$ on one or the other side of those relations, so as to formally

¹Under gauge symmetry breaking, θ could be identified with whatever particular value the θ_W has.

²The basic scales, like the L_o , are carried by the frames. All other quantities must have either derivable or inherently independent scale (like the L_1 , to be identified with the EW), or be scaleless as with, e.g., all entries in (1.3). In the scaleless coupling $g = \sqrt{2}\kappa/L_o$, the denominator de-scales Ξ_I^α (circumstantially scaled by L_o in (2.1)) and κ , identifiable as the $\sqrt{8\pi G_N}$ gravitational coupling, provides the missing scale.

also define \bar{e}^A in terms of e^A etc., with the *same* components of $g[\Xi \cdot \mathcal{A}]$. This reveals an underlying *tilt invariance*, whereby tensorial components like Ξ_I^m , \mathcal{A}_ν^I , $[\Xi \cdot \mathcal{A}]_\nu^m$, $[\Xi \cdot \mathcal{A}]^m$ etc remain the same in either of (\bar{e}^M, \bar{E}_N) , (e^M, E_N) . This is due to the ‘diagonality’ of $e^\mu = \bar{e}^\mu$ $E_n = \bar{E}_n$ and survives the generalized excitation of \bar{e}^μ to $e^\mu = e_\mu^\mu dx^\mu$, etc., to be introduced later-on by (4.1). This tilt invariance can simplify calculations considerably; its members also include volume elements like $\bar{\varepsilon} = \varepsilon$ and derivations from $\bar{E}_n = E_n$, but *not* ordinary partial derivatives from $\bar{E}_\mu \neq E_\mu$. For the latter kind, by excitation of ∂_μ under the tilt of the frames in (2.2), a rigorous gravito-EW ‘minimal-coupling’ rule can be uncovered as

$$\bar{E}_\mu \rightarrow E_\mu = \bar{E}_\mu - g[\Xi \cdot \mathcal{A}]_\mu \implies \partial_\mu \rightarrow \partial_\mu - g(\xi_i \mathcal{A}_\mu^i \sin \theta + \xi_4 \mathcal{A}_\mu^4 \cos \theta). \quad (2.4)$$

To proceed with the calculation of the \mathcal{L}_{HEC} Lagrangian, we first note that we have not yet arrived at $\mathcal{M}^8(e^A, \gamma_B^A)$ of (1.2), because we have not yet excited the connection. Accordingly, we will employ an asterisk $*$ on our not-yet-excited intermediate γ_N^{*M} , which, of course, has changed anyway from the $\bar{\gamma}_N^M$ value, due to the $\delta \bar{e}^A$ excitation, so its Christofel part is identical to the one involved in the standard Kaluza ansatz. The calculation towards the intermediate $\mathcal{L}_{\text{HEC}}^* = \mathcal{R}^*$ (modulo surface terms) involves the basic preliminary result

$$de^m = g[\Xi \cdot \mathcal{F}]^m - \frac{1}{2L_o} \delta_i^m \epsilon_{jk}^i e^j \wedge e^k, \quad [\Xi \cdot \mathcal{F}]^m := \Xi_i^m \mathcal{F}^i \sin \theta + \Xi_4^m \cos \theta \mathcal{F}^4, \quad (2.5)$$

with the gauge-field strength \mathcal{F} as defined below. Its kinetic term emerges in $\mathcal{L}_{\text{HEC}}^*$ as in the standard treatment, while the rest, formally included in [GR + GEW terms] sector, relates to gravity and torsion. In view of the $\bar{\mathcal{R}} = 0$ result in $\bar{\mathcal{M}}^8$ there will be no effective cosmological constant from reduction to 4 spacetime dimensions. These aspects of the

$$\mathcal{L}_{\text{HEC}}^* = [\text{GR} + \text{GEW terms}] - \frac{1}{2} \kappa^2 \mathcal{F}^2, \quad \mathcal{F}^I := d\mathcal{A}^I + \frac{1}{2} g \sin \theta \delta_i^I \epsilon_{jk}^i \mathcal{A}^j \wedge \mathcal{A}^k, \quad (2.6)$$

Lagrangian will not be influenced by the mentioned generalization in (4.1), and our remarks following (2.1) do apply, of course, to the gauge-invariant environment established by (2.6). The slicing angle θ therein is fully redundant (with a trivial re-definition of $g \sin \theta$) and we could have dispensed with it *and* the Killing vectors altogether. In fact, it would have been easier to arrive at (2.6) by simply employing, instead of (e^M, E_N) , a left-invariant frame. For that, we could have simply used $L_o \delta_I^m \mathcal{A}^I$ instead of $[\Xi \cdot \mathcal{A}]^m$ in (2.2), and then proceed as usual to verify the claim. We conclude that, as long as the gauge symmetry is respected, any particular slicing of the torus is as good as any other, so the slicing angle θ must drop out in a symmetric environment, and it does. Indeed, the θ -dependence of the $L_o/\sin \theta$ and $L_o/\cos \theta$ radii in the orthonormality relations between the Killing vectors in (2.1) works in conjunction with the standard choice in (2.3) for the dependence of $[\Xi \cdot \mathcal{A}]$ on θ , so the slicing angle is precisely canceled out. However, this seemingly ‘useless’ involvement of θ will prove crucial and irreplaceable for the implementation of the upcoming gauge-symmetry breaking by the KA2 approach, as we’ll see in the next section.

3 Excitation of the torsion to arrive at $\mathcal{M}^8(e^A, \gamma_B^A)$

For completion of the KA2 approach in (1.2), our last step involves the excitation of the torsion to $T^M = \bar{T}^M + \delta\bar{T}^M$ and the induced $K_N^M = \bar{K}_N^M + \delta\bar{K}_N^M$. Both excitations are linear in \mathcal{A}^I (as with (2.2) for $\delta\bar{e}^M$) and linearly related among themselves (by (1.5) etc) as

$$\delta\bar{K}_{MNP} = -\frac{1}{2}(\delta\bar{T}_{MNP} + \delta\bar{T}_{NPM} - \delta\bar{T}_{PMN}) . \quad (3.1)$$

However, before we proceed with the calculation of this (proportional to \mathcal{A}^I and scaled by the mentioned L_1) $\delta\bar{T}^M$, we must make sure that this excitation of the connection is indeed independent, namely that the field-content of \mathcal{A}^I has not been already exhausted towards the $\delta\bar{e}^M$ tilt of the frames in (2.2). As we'll see in the last section, unspent degrees of freedom in \mathcal{A}^I do survive in this case and they are *precisely* enough to accommodate the transverse degrees of freedom of the EW gauge bosons. We can easily find that *any* general connection γ_N^M , enlarged to $\gamma_N^M + K_N^M$, induces the $K^{MPN}K_{NPM} - K^{MP}_M K^N_{PN}$ contribution (modulo surface terms) to an accordingly enlarged \mathcal{L}_{HEC} Lagrangian. Thus, when the intermediate connection γ_N^{*M} is excited to $\gamma_N^M = \gamma_N^{*M} + \delta\bar{K}_N^M$, the intermediate $\mathcal{L}_{\text{HEC}}^*$ in (2.6) will be accordingly elevated to final form with quadratic-in- $\delta\bar{K}$ terms, as

$$\mathcal{L}_{\text{HEC}} = [\text{GR} + \text{GEW terms}] - \frac{1}{2}\kappa^2 \mathcal{F}^2 + \delta\bar{K}^{APB} \delta\bar{K}_{BPA} - \delta\bar{K}^{MP}_M \delta\bar{K}^N_{PN} . \quad (3.2)$$

Due to the implicit presence of quadratic-in- \mathcal{A}^I terms (in the two last ones on the r.h.s.), we have already lost the $SU(2) \times U(1)$ left invariance which had previously covered the entire $\mathcal{L}_{\text{HE}}^*$ in (2.6), so gauge-symmetry breaking has already occurred in the \mathcal{L}_{HEC} of (3.2), as a result of the excitation $\delta\bar{T}^M$ of the connection.

To replace \sim with precise equality in $\delta\bar{T}^M = \frac{1}{2}\delta\bar{T}_{NP}^M e^N \wedge e^P \sim L_1^{-1} \mathcal{A}^I$, we note that the missing tensorial factor on the r.h.s. must: depend only on the Killing vectors Ξ_J , have exactly one free group-index I (to saturate the free-one on \mathcal{A}^I) and balance the rest of the free indices in that relation. To accomplish that, we must exploit the already-installed breaking of gauge invariance in (3.2), in the sense that there exists an unknown but *specific* angle θ_W , by which the $S^3 \times S^1$ torus can be viewed as already sliced. By fixing (in-retrospect agreement with standard convention) the $I=3$ direction, we can introduce a fixed mixing as $\Xi_{(W)}^b \sim (\Xi_3^b \sin \theta_W + \Xi_4^b \cos \theta_W)$ without loss of generality. This $\Xi_{(W)}^b$ times a Ξ_I^p for the required free index I (and antisymmetry for torsion) accommodates fully and precisely all the requirements on our tensorial factor as $\Xi_{(W)}^{[b} \Xi_I^{p]}$, so the final result is *unique* as

$$\delta T^p = \frac{g}{L_1} \eta^{\rho\sigma} \eta_{bm} \eta_{pn} \Xi_{(W)}^{[b} \Xi_I^{p]} \mathcal{A}_\sigma^I e^m \wedge e^n, \quad \Xi_{(W)}^b := \frac{1}{\sqrt{2}L_0} (\Xi_3^b \sin \theta_W + \Xi_4^b \cos \theta_W) . \quad (3.3)$$

Thus, by (3.1), the only non-vanishing independent components of $\delta\bar{T}^M$ and $\delta\bar{K}_N^M$ are

$$\frac{1}{2}\delta\bar{T}^{\mu bp} = -\delta\bar{K}^{\mu bp} = \delta\bar{K}^{bp\mu} = \frac{g}{L_0 L_1} \eta^{\mu\nu} \Xi_{(W)}^{[b} \Xi_I^{p]} \mathcal{A}_\nu^I . \quad (3.4)$$

To find explicitly the mass term already present in (3.2), we may re-write the latter as

$$\mathcal{L}_{\text{HEC}} = [\text{GR} + \text{GEW terms}] - \frac{\kappa^2}{2} \mathcal{F}^2 - \kappa^2 M_{IJ} \mathcal{A}_\mu^I \mathcal{A}_\nu^J \eta^{\mu\nu}, \quad (3.5)$$

wherefrom, by the tracelessness of contorsion from (3.4), we can read-out the identification

$$\kappa^2 M_{IJ} \mathcal{A}_\mu^I \mathcal{A}_\nu^J \eta^{\mu\nu} = -\delta K^{APB} \delta K_{BPA}. \quad (3.6)$$

The straightforward substitution of (3.4) in (3.6) quantifies the mass matrix etc., as

$$M_{IJ} = (\text{L}_o \text{L}_1)^{-2} [(\Xi_{(W)})^2 \Xi_I^b \Xi_J^p \eta_{bp} - (\Xi_{(W)} \cdot \Xi)_I (\Xi_{(W)} \cdot \Xi)_J], \quad (3.7)$$

$$(\Xi_{(W)} \cdot \Xi)_I := \Xi_{(W)}^b \Xi_I^p \eta_{bp} = \frac{\text{L}_o}{\sqrt{2}} \left(\frac{\sin \theta_W}{\sin^2 \theta} \delta_{3I} + \frac{\cos \theta_W}{\cos^2 \theta} \delta_{4I} \right), \quad (3.8)$$

where, having used the orthonormality relations from (2.1), we must now set $\theta = \theta_W$. With $(\Xi_{(W)})^2 = 1$, as normalized in (3.3), we may express the mass-term in (3.7) as

$$M_{IJ} = (\text{L}_1 \sin \theta_W)^{-2} \left[\eta_{ij} \delta_I^i \delta_J^j - \frac{1}{2} (\delta_I^3 \delta_J^3 + \tan^2 \theta_W \delta_I^4 \delta_J^4 + \tan \theta_W (\delta_I^3 \delta_J^4 + \delta_I^4 \delta_J^3)) \right], \quad (3.9)$$

or, equivalently, in the more conventional matrix notation, as

$$M_{IJ} = (\text{L}_1 \sin \theta_W)^{-2} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & -\frac{1}{2} \tan \theta_W \\ 0 & 0 & -\frac{1}{2} \tan \theta_W & \frac{1}{2} \tan^2 \theta_W \end{pmatrix}. \quad (3.10)$$

Either

$$\Delta^I{}_J = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -\cos \theta_W & \sin \theta_W \\ 0 & 0 & +\sin \theta_W & \cos \theta_W \end{pmatrix} \text{ or } \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 & 0 \\ -i/\sqrt{2} & i/\sqrt{2} & 0 & 0 \\ 0 & 0 & -\cos \theta_W & \sin \theta_W \\ 0 & 0 & +\sin \theta_W & \cos \theta_W \end{pmatrix} \quad (3.11)$$

diagonalizes M_{IJ} to its eigenvalues as

$$M^I{}_J = (\text{L}_1 \sin \theta_W)^{-2} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{2} (\cos \theta_W)^{-2} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} m_W^2 & 0 & 0 & 0 \\ 0 & m_W^2 & 0 & 0 \\ 0 & 0 & \frac{1}{2} m_Z^2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad (3.12)$$

so the gauge-boson masses are post-dicted as $m_W = (\text{L}_1 \sin \theta_W)^{-1}$, $m_Z = (\text{L}_1 \sin \theta_W \cos \theta_W)^{-1}$, and the $\rho := m_W^2 / (m_Z \cos \theta_W)^2$ parameter as $\rho = 1$. For the physical gauge bosons, actually read off (3.11) as $W^\pm = (\mathcal{A}^1 \mp i \mathcal{A}^2) / \sqrt{2}$, $Z = -\cos \theta_W \mathcal{A}^3 + \sin \theta_W \mathcal{A}^4$, $B = \sin \theta_W \mathcal{A}^3 + \cos \theta_W \mathcal{A}^4$, the \mathcal{L}_{HEC} of (3.5) takes on its standard expression, with the mass term therein as

$$\kappa^2 M_{IJ} \mathcal{A}_\alpha^I \mathcal{A}_\beta^J \delta^{\alpha\beta} = \kappa^2 (m_W^2 W^+ W^- + \frac{1}{2} m_Z^2 Z^2). \quad (3.13)$$

4 Discussion

In spite of its simplicity, the KA2 approach by (1.2) vs the standard (1.1) offers surprisingly far-reaching results and relevance to fundamental issues, hitherto unrelated. We will briefly expand toward some of them, after some clarifications and pending completions. As anticipated (in view of the $\bar{\mathcal{R}}=0$ result in $\bar{\mathcal{M}}^8$), in order to have a general curvature scalar \mathcal{R} present in the [GR + GEW terms] general relativistic and gravito-EW sector in (3.5), we must allow for a generalized excitation of \bar{e}^μ to e^μ in (2.2), now re-defined in terms of the vierbeins e^μ_ρ and inverse E^ρ_ν (namely with $e^\mu_\rho E^\rho_\nu = \delta^\mu_\nu$) as

$$e^A = e^\mu_\mu dx^\mu \delta_\mu^A + (\bar{e}^m + g[\Xi \cdot \mathcal{A}]^m) \delta_m^A \iff E_B = (E^\nu_\nu \partial_\nu - g[\Xi \cdot \mathcal{A}]_\nu) \delta_B^\nu + \bar{E}_n \delta_B^n. \quad (4.1)$$

When we set out for the simplest non-trivial implementation of the KA2 approach, we initially anticipated to utilize one of the $\bar{\mathcal{M}}^7 = \bar{\mathcal{B}}_T^4 \times \mathbb{R}^3$ or $\bar{\mathcal{M}}^7 = \bar{\mathcal{B}}_M^4 \times \mathbb{R}^3$ ground states, instead of the finally employed $\bar{\mathcal{M}}^8 = \bar{\mathcal{B}}_M^4 \times S^1 \times \mathbb{R}^3$ in (1.2). The S^1 factor therein is not merely a ‘spectator’, but it is rather imposed by *augmendability* under KA2, as we’ll see. By general considerations [1], the 7-dimensional \mathcal{M}^7 proper vacuum would have involved a total of $7(7-3)/2=14$ independent states in its \mathcal{L}_{HEC} ; the piecemeal count of the 2 graviton states in \mathcal{M}^7 plus the $SU(2)$ scalar and *massless* gauge boson states as $2+3(3+1)/2+2\cdot 3$ would have again given us precisely 14. This means that there would be no field-content left in the corresponding \mathcal{A}^T for an independent variation of the torsion. Thus, for our \mathcal{M}^7 cases, we would end-up with a gauge-invariant \mathcal{L}_{HEC} and no mass-term, hence with a KA2 approach redundant to the standard one. A non-redundant KA2 is achieved with the minimally augmented $\mathcal{C}^5 = \mathcal{B}_M^4 \times S^1$ for the $\bar{\mathcal{M}}^8 = \bar{\mathcal{B}}_M^4 \times S^1 \times \mathbb{R}^3$ ground state in (1.2). Indeed, in this case, the general count of $8(8-3)/2=20$ states for \mathcal{M}^8 is not quite matched by the piecemeal count of states, *again* for massless (now $SU(2) \times U(1)$) gauge bosons, because $2+(3(3+1)/2+1)+2\cdot(3+1)=17$. The surviving 3 degrees of freedom in \mathcal{A}^T have been precisely enough to generate (by independent excitation of the torsion) the 3 transverse states of the mass term (3.13) in the Lagrangian (3.5). Thus, the notion of *augmendability* by KA2 emerges as complementary to the requirement for spontaneous compactification. As a result, $\mathcal{C}^5 = \mathcal{B}_M^4 \times S^1$ is augmendable under KA2, but the $\mathcal{C}^4 = \mathcal{B}_{\text{IX}}^4$ is not.

We are now better equipped to sum-up our results as follows.

(i) The gravitational and electroweak sectors have emerged in elegant *hierarchy* in a \mathcal{L}_{HEC} of the form (3.5), in terms of the (identifiable as gravitational) coupling κ and the EW scale L_1 . The gravitational interaction is an effective one at scales $L \gg L_o$ and its frames e^μ , as defined by (4.1), are *in principle* calculable via Einstein’s equations. The latter will follow from any Lagrangian of the type (3.5), after the fashion of ‘electrovac equations’ and a ‘minimal-coupling’ rule, now elevated to gravito-EW vacuum equations and (2.4), respectively.

(ii) The *higgsless* emergence of the EW gauge-boson masses is fully calculable by (3.7-3.13), although the numerical value of θ_W would be calculable only with the employment of the $\tilde{\mathcal{B}}_T^4$ of the Taub string (instead of the $\tilde{\mathcal{B}}_M^4$) in (1.2), via the set of the radii of its squashed S^3 , the $(1/\sqrt{3}, 1/\sqrt{3}, \sqrt{\pi/2 - 1})$ in units of L_o [2]. The mass term in (3.5) has been produced by the geometry via excitation of the effective torsion, actually the only geometric element which could protect against mathematical singularities, if masses were to be added by-hand.

(iii) By KA2, if mathematical singularities (but not *physical* ones) were to be excluded from physical spacetime, the fundamental rôle of gravity in the dynamics of vacuum geometry is being conceded to the remaining fundamental interactions. Gravity does retain all its geometric aspects, but the dimensionful coupling κ is now its only relation to Planck scale. At or close to that scale, where everything is *actually* part of a true proper vacuum, the meaning of a gravitational coupling is empty anyway. At the intermediate regime, where all other interactions are quantized (say, very widely around L_1), gravity would again be in a $L \gg L_o$ environment, so it would remain classical there, as in ordinary 4D classical regime. It would then follow that gravity can only stand as an *effective* interaction or classical field in 4D, and as such it would have to be excluded from quantization.

Thus, by our findings via the KA2 approach, we may conjecture an augmentable \mathcal{C}^n to adjoin the strong interaction towards a standard model 2, which already includes gravity. In view of the reasonable $n \leq 7$ requirement, this would also offer the option of an analogous re-orientation in supergravity [1].

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References

- [1] M.J. Duff, B.E.W. Nilsson and C.N. Pope, *Phys. Rep.* **130** (1986) 1.
J.M. Overduin and P.S. Wesson, *Phys. Rep.* **283** (1997) 303.
- [2] N.A. Batakis, arXiv:1203.0881 [gr-qc], submitted to *Class. Quantum Grav.*
- [3] A. Trautman, *Symp. Math.* **12** (1973) 139.
- [4] S. Helgason, *Differential Geometry and Symmetric Spaces* (1962, Ac. Press, New York).
- [5] M. Ryan and L. Shepley, *Homogeneous Relativistic Cosmologies*, (P.U.P., N.J., 1975).
- [6] C.W. Misner, *Phys. Rev. Lett.* **22** (1969) 1071.